

# Bills paradox

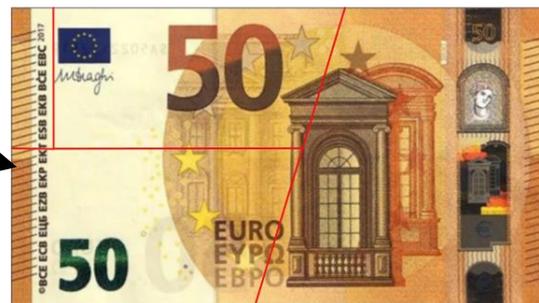
## Topic/Learning objectives of exhibit:

Geometry, strategy, solving problems skills, communication.

## List of Materials Required:

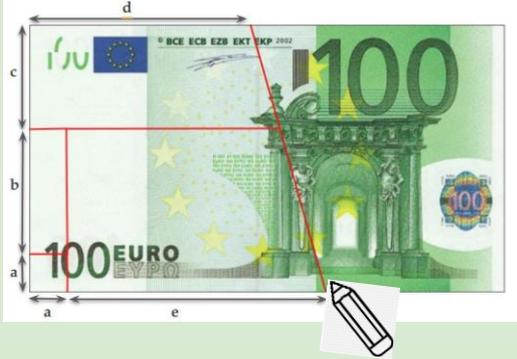
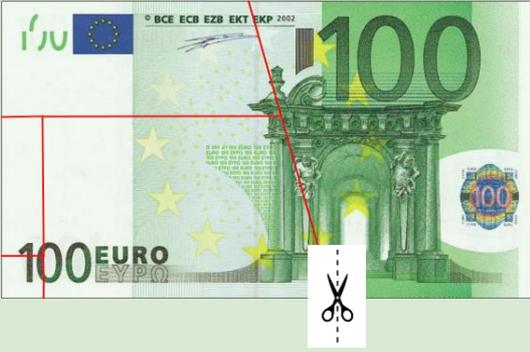
- Thick paper (>150g/m<sup>2</sup>) DIN A4
- A regular Printer
- Pencil
- scissors
- A ruler
- Glue

## Step-by-step Construction



Estimated Time: 45'

## FIRST PARADOX

<p>Step 1 Print the bills.</p>	<p>Get the file and print the 2 bills (100€ &amp; 50€) in the same DIN A4 sheet.</p> <p>File: <a href="https://drive.google.com/file/d/1Hlse055uQYuttbHhHHcBSQroNqFwJcHN/view?usp=share_link">https://drive.google.com/file/d/1Hlse055uQYuttbHhHHcBSQroNqFwJcHN/view?usp=share_link</a></p>	
<p>Step 2 Draw the lines.</p>	<p>For Bill Dimensions: 15cm x 8,5cm</p> <p>Take a = 1,1 cm b = c = 3,7 cm d = 7 cm e = 8,1 cm</p> <p>With the help of the ruler and a pencil, draw the lines on the 100€ bill.</p>	
<p>Step 3 Cut the pieces.</p>	<p>With the scissors, cut along the red lines to get 5 pieces:</p> 	

Step 4  
Make the pieces for the 50€ bill

Now, flip the pieces back and assemble them correctly on the €50 bill in order to draw the lines of the pieces, i.e, use the pieces of the 100€ bill to draw the lines for the 50€ bill.  
TIP: start from the left part of the bill.



Step 5  
Cut the pieces of the 50€ Bill

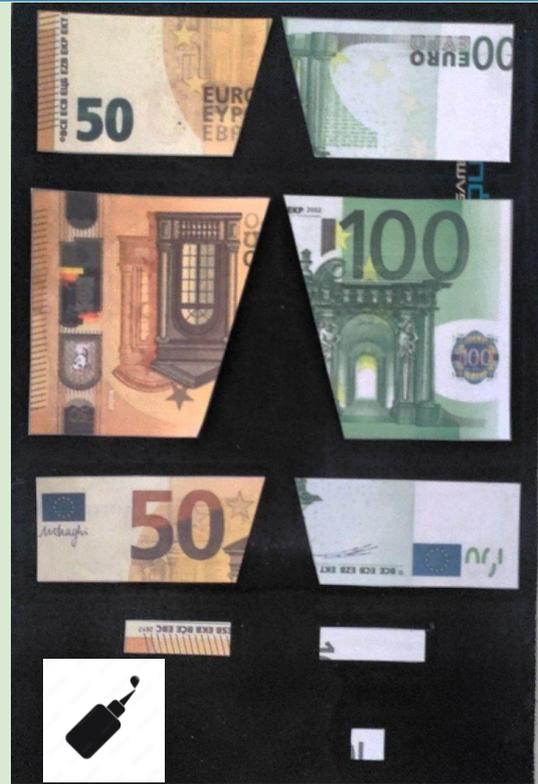
With scissors, cut the 50€ bill and then cut along the lines to get 4 pieces.



Step 6  
Glue the pieces together

Now, you have 5 pieces from 100€ and 4 pieces from 50€.

Glue the pieces in couples with the same form, from the back side, to get 4 pieces: each piece should have one side with a part of the 100€ and the other of the 50€. There's one piece from the 100€ without "partner" (the small square)



Step 7  
Cut the excess paper off

In the biggest piece, there should be a small rectangle of paper that "exits" from the 100€ piece side: you have to cut it off. This area is exactly the same area of the disappeared square.

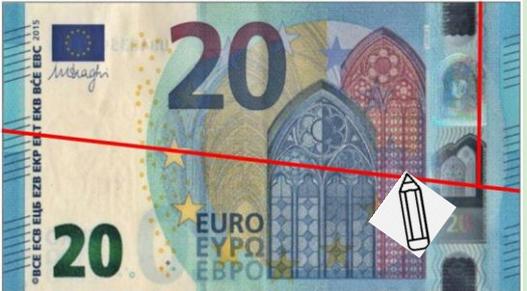
The rest of the pieces should match perfectly



<p>Step 8 Done it.</p>	<p>Now you have the first bill paradox made.</p>	
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## SECOND PARADOX:

<p>Step 1 Print the bills.</p>	<p>Get the file and print the 2 bills (50€ &amp; 20€) in the same DIN A4 sheet.</p> <p>File: <a href="https://drive.google.com/file/d/123_jvymAl6fRQn0gapSTb2zDEX8bfbJq/view?usp=share_link">https://drive.google.com/file/d/123_jvymAl6fRQn0gapSTb2zDEX8bfbJq/view?usp=share_link</a></p>	
<p>Step 2 Draw the lines.</p>	<p>For Bill Dimensions: 15cm x 8,5cm</p> <p>Take <math>a = b = 4,5 \text{ cm}</math> <math>c = 1 \text{ cm}</math></p> <p>With the help of the ruler and a pencil, draw the lines on the 50€ bill.</p>	

<p>Step 3 Cut the pieces.</p>	<p>With the scissors, cut along the red lines to get 4 pieces:</p>	
<p>Step 4 Make the pieces for the 20€ bill</p>	<p>Now, flip the pieces back and “assemble” them correctly on the €20 bill in order to draw the lines for cutting along the pieces, i.e., use the pieces of the 50€ bill to draw the lines on the 20€ bill</p>	
<p>Step 5 Cut the pieces of the 20€ Bill</p>	<p>With scissors, cut the cut along the lines of the 20€ bill to get 3 pieces.</p>	

<p>Step 6 Glue the pieces together</p>	<p>Now, you have 4 pieces from 50€ and 3 pieces from 20€.</p> <p>Glue the pieces in couples with the same form, from the back side, to get 3 pieces: each piece should have one side with a part of the 50€ and the other of the 20€.</p> <p>There's one piece from the 50€ without "partner" (the small square)</p>	
<p>Step 7 Cut the excess paper off</p>	<p>There should be some paper that "exits" from the 20€ pieces side: you have to cut them off. This area is exactly the same area of the disappeared square.</p> <p>The pieces should match perfectly.</p>	
<p>Step 8 Done it.</p>	<p>Now you have the second bill paradox made.</p>	

## Observations

It is extremely important to take the measures perfectly and be very precise in drawing the lines and cutting the pieces.

If the bills have a different size, the measures of the pieces have to be changed, but you must keep this relationship between the sides:

For the 100€ - 50€

$$b=c$$

$$a+d=e$$

For the 20€ - 50€

$$a=b$$

(the rest of the sides can be as desired)