In The Balance - Scale Balance

Materials

Exhibit board, 11 modular weights, 1 fake weight, graduated roman weighing scale

Brief description

This activity intends to both challenge logical reasoning as well as understand proportions. It can lead into exercises in logic reasoning and algebra. The exhibit gives the participant an opportunity to reason and interact with a scale. Two problems are put to puzzle and challenge participants.The scales are both plate and rudimentarily graduated. This offers itself to problems that can be implemented either by adding or subtracting weights from the graduated scale or by directly comparing two quantities on the plates.

Assembly

Design of all the pieces

The centerpiece of the exhibit is a graduated plate weighing scale. The exhibit board gives two challenges to be solved resorting to the scales or reasoning. This is accompanied by modular unit weights. There should be one weight with less mass for the first problem. The weights are modular and can be used individually or assembled to be hung in a group from the beam.

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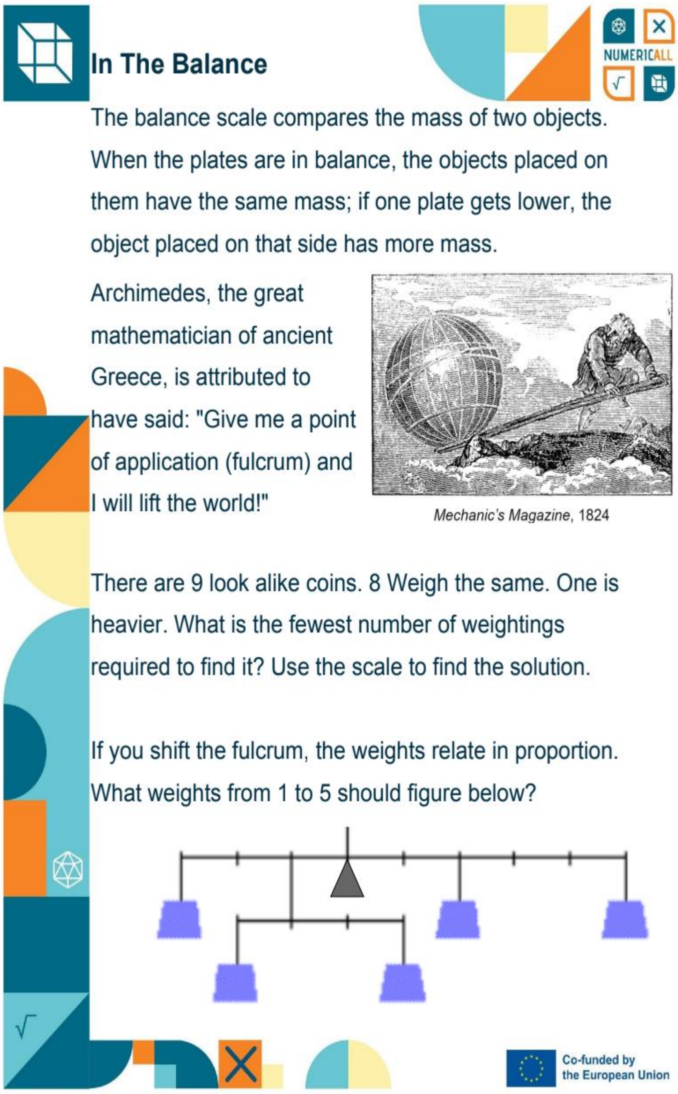
Assembly

Together with the problem set the scales and weights should be readily and openly placed on a table. The 3D model can be printed and put together, for instructions and other implementation ideas please refer to the DIY documentation.

The Board (DINA3)



In The Balance

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The balance scale compares the mass of two objects. When the plates are in balance, the objects placed on them have the same mass; if one plate gets lower, the object placed on that side has more mass.

Archimedes, the great mathematician of ancient Greece, is attributed to have said: "Give me a point of application (fulcrum) and I will lift the world!"

Mechanic's Magazine, 1824

There are 9 look alike coins. 8 Weigh the same. One is heavier. What is the fewest number of weightings required to find it? Use the scale to find the solution.

If you shift the fulcrum, the weights relate in proportion. What weights from 1 to 5 should figure below?

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Other Options

Other than the suggested problems on the exhibit board various others can be stated. Exploration of the properties of equilibria can be explored, "What has to happen in order to balance the roman weighing scale?", "If this object weighs 10 g, how much do others weigh in comparison?"

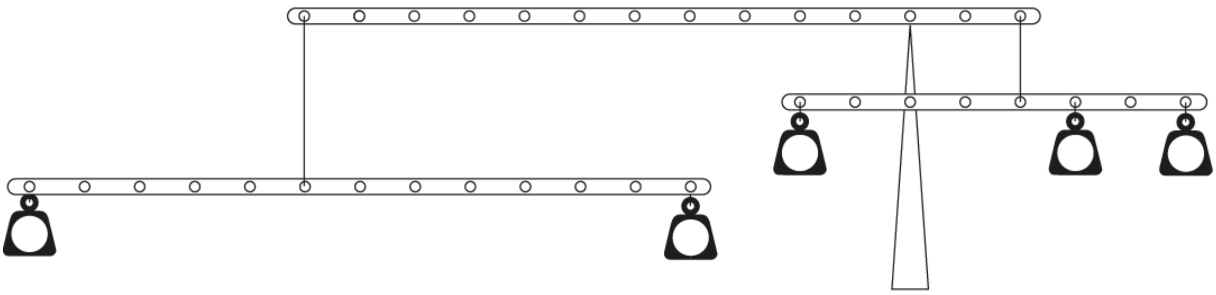
Below we state two additional problems to be suggested by a monitor or in a learning environment.

Comparative Weightings Problems

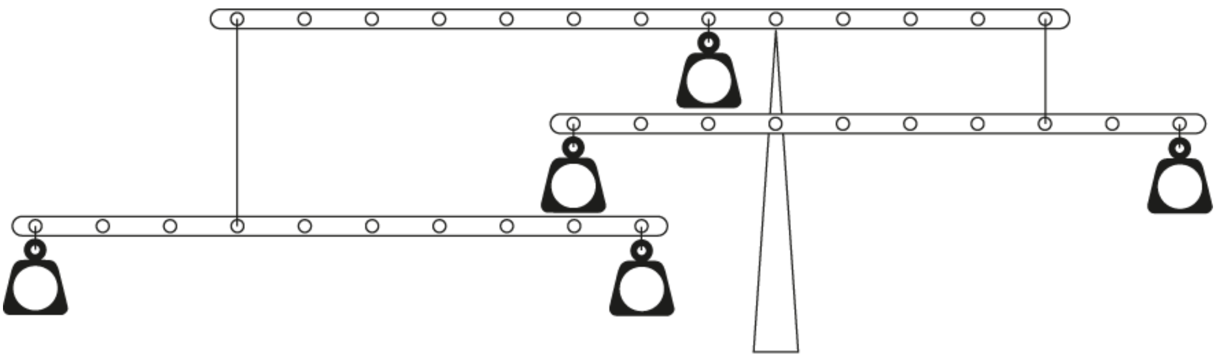
1. "Given 5 coins out of which one coin is lighter. In the worst case, what is the minimum number of weightings required to figure out the fake coin?"
2. "This time we are given 3 coins. If one is different, we don't know whether it is heavier or lighter than the others. What is the minimum number of weighings needed to determine if there is a false coin and which one it is?"

Beam Equilibrium Problems

3. Using the Weights with Values between 1-28 find a solution for the following equilibrium:



4. Using the Weights with Values between 1-20 find a solution for the following equilibrium:



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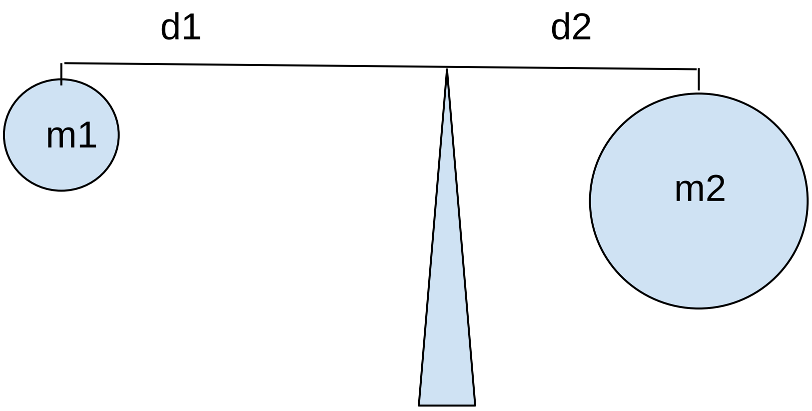
Explanation

A lever is a beam connected to ground by a hinge, or pivot, called a fulcrum. The lever is a movable bar that pivots on a fulcrum attached to a fixed point. The lever operates by applying forces at different distances from the fulcrum, or a pivot.

As the lever rotates around the fulcrum, points farther from this pivot move faster than points closer to the pivot. Therefore, a force applied to a point farther from the pivot must be less than the force located at a point closer in, because power is the product of force and velocity. This is known as the law of the lever (see image below).



This proportionality allows for multiple comparisons between plates at equal distances on which different weights are placed, or weights placed closer or further from the leaver (the weight of the beam is disregarded here).
When comparing weights, resources to decision trees makes the visualization and solution of the problems more evident. In the Beam equilibrium problems we can resort to the use of algebraic formulation to aid with calculations necessary to find a solution.

Solutions to the problems of the exhibit board:
Solution to the comparative weighting problem from the exhibit board:
The minimum number of weightings is 2. The coins are grouped in trios. When weighing two of these groups there are two options: they balance or one is heavier. If they balance, this means that the heavier coin must be in the other group. The next weighting should compare two coins from that group. If one is heavier the next weighting should take two coins from the heavier group. In either case two coins from a group get compared. Either the next weighting reveals the heavier coin or, if the result is balanced, the coin left out from the group must be the the one left out
Solution to the beam equilibrium problem from the exhibit board:

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The equilibrium described by the image can be put as two equalities, assigning letters to the weights:

A = 2B (1) and 3C + 1(A + B) = 2D + 5E (2), where A, B, C, D, E have to be integer solutions from 1 to 5. The solutions for (1) are either (2, 1) or (4, 2). If we choose the first solution the left side of (2) will be a multiple of 3. Using the remaining number it is impossible to satisfy this condition. So we know that A = 4 and B = 2. The left side still is a multiple of 3, but the available numbers are enough to satisfy this having D = 5 and E = 1 we get 2 * 5 + 1 * 5 = 3 * 5 = 15 and on the other side only C = 3 is an option resulting in 3 * 3 + 2 + 4 = 15. Solving the problem. (A, B, C, D, E) = (4, 2, 3, 5, 1).

Additional Problems Solutions:

1. Is a simpler version of the exhibit board; 2 weightings is the correct answer to guarantee finding the coin, by the same argument;
2. Two weightings is the correct answer. Although there are less coins, not knowing if the coin is heavier or not requires comparing two different pairs of coins.
3. From left to right let the weights be in alphabetical order. The solution is (A, B, C, D, E) = (7, 5, 24, 15, 27)
4. The solution is (A, B, C, D, E) = (3, 4, 5, 8, 12)

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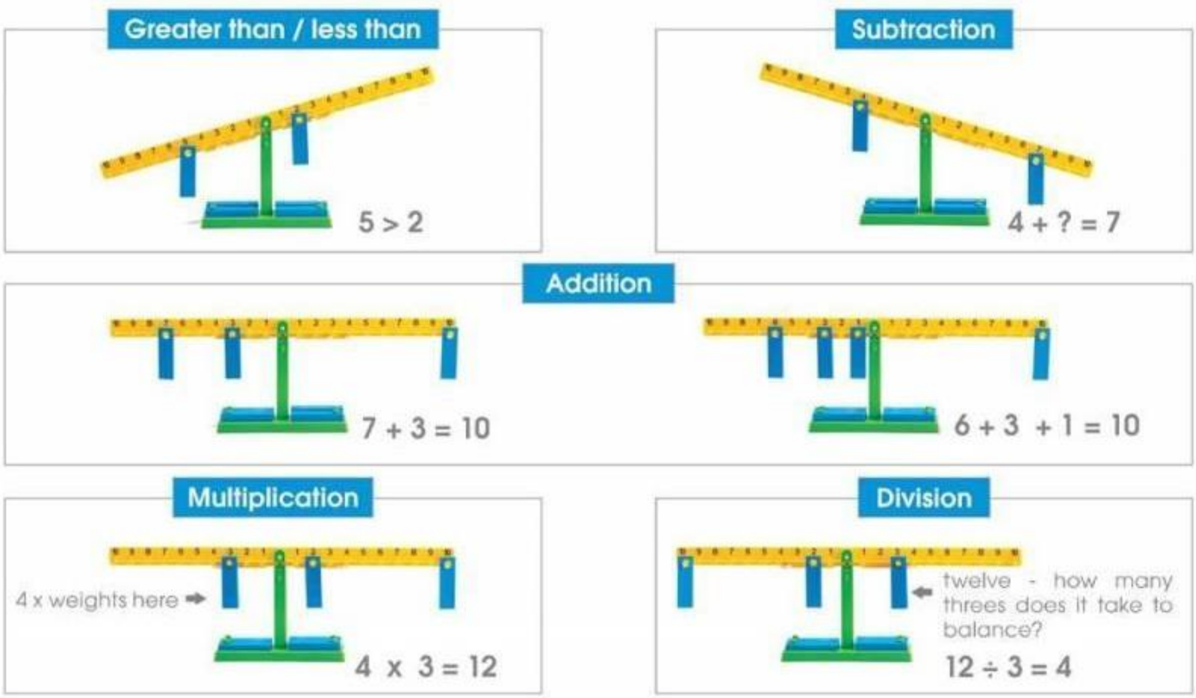
Competencies

- Logical deduction
- Proportionality
- The notion of “unit weight” and comparison-based deduction of the proportions of the pieces without using measuring tools, only the balance.
- Computing the weight of some objects.
- Equilibrium as a notion of equality.
- Mental arithmetic: products and additions.
- Praxis in trial-and-error methodology.

Observations

The scales might need calibration, this can easily be done with duct tape until the beam without any weights is balanced.

Some mathematical concepts like equalities and inequalities are easily visualized using the scales. A visual example below:



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For 3d Printers (If applicable)

The DIY can be found as well as any 3D printable files at:

https://drive.google.com/drive/folders/1F8JySKT56nZZd0oEDAV501hNzM6W4pQx?usp=sharing

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